ADAPTIVE CONTROL AND SYNCHRONIZATION OF MEMRISTOR-BASED HYPERCHAOTIC SYSTEM WITH UNKNOWN PARAMETERS

Supported by tetfund

O. I. Olusola*[1], V. C. Ozebo[1], O. A. Ajayi[1] and J. A. Laoye[2]

[1]Department of Physics, University of Lagos, Akoka, Lagos State, Nigeria

[2]Department of Physics, Olabisi Onabanjo University, Ago-Iwoye, Ogun State, Nigeria

*Corresponding author: olasunkanmi2000@gmail.com

Received: December 06, 2016 **Accepted:** February 05, 2017

Abstract: This paper studies the control and synchronization of hyperchaotic memoristive circuits with unknown parameters using adaptive control approach. The designed adaptive nonlinear controllers globally control and synchronize two identical hyperchaotic memoristive systems evolving from different initial conditions. The adaptive approach is suitable for addressing uncertainties in systems parameters and environmental disturbances that can badly affect control and synchronization performance. The effectiveness and feasibility of the designed nonlinear controllers is verified and demonstrated numerically.

Keywords: Adaptive approach, chaos control, hyperchaotic, memoritive system, synchronization

Introduction

The year 1990 marks a major turning point in the study of nonlinear dynamical systems when the idea of synchronization of chaotic systems was presented by Pecora and Carroll (Pecora and Carroll, 1990). Since then, the phenomenon and its application in secure communication has attracted intensive research attention (Cuomo *et al*., 1993; Ying and Chua, 1997; Sundar and Minai, 2000; Boutayeb *et al*., 2002; Guan *et al*., 2002; Feki and Ma *et al*., 2003; Pan *et al*., 2010; and Zhang *et al*., 2010). Chaos synchronization had formed interdisciplinary applications in varieties of field of study including time series analysis, modelling cardiac rhythm and brain activity, and earth quake dynamics (Yang and Duan, 1998; Pikovsky *et al*., 2001; Eisencraft *et al*., 2012; Ren *et al*., 2013; Aguilar-Lopez *et al*., 2014; Filali *et al*., 2014), these have provided the driving force for the enormous effort being devoted to different ways of achieving chaos synchronization in different systems.

Generally, two interacting chaotic systems with state space variables $x_1(t)$ and $y_1(t)$ becomes completely or identically synchronized if the synchronization manifold $x_1(t) = y_1(t)$ exists and the condition $\lim_{t\to\infty}\|x_1(t) - y_1(t) = 0\|\ \forall t \geq 0$ is satisfied (Pecora and Carroll, 1990). Other types of synchronization that have been widely studied and reported in literature include generalized synchronization (Yang and Duan, 1998; and Wang and Guan, 2006), phase synchronization (Michael *et al*., 1996; Ho *et al*., 2002; Di *et al*., 2005), lag synchronization (Di *et al*., 2005), projective synchronization (Mainieri and Rehacek, 1999), anti-synchronization (Zhang and Sun, 2004) etc. Also, a wide variety of methods for synchronization and control of chaotic/hyperchaotic systems have also been proposed in recent years, such as linear state feedback control method (Olusola *et al*., 2009), adaptive control method (XingyuanWang and YaqinWang, 2011), impulsive control method (Ying and Chua, 1997), Observer-based method (Liu *et al*., 2009) global synchronization method (Zhang *et al*., 2010) and so on.

In recent years, hyperchaotic systems have attracted huge body of knowledge in nonlinear science. Hyperchaotic system is characterized with more than one positive Lyapunov exponent which generates more complex dynamics than the low dimensional chaotic systems. For instance, it has been shown that hyperchaotic systems are more effective for secure communication and the presence of more than one positive Lyapunov exponent clearly improves the security of the communication scheme (Elabbasy *et al*., 2006). In the present paper, we examine control and synchronization of memristor-based hyperchaotic systems via extended adaptive control approach. This approach is significant and of vital importance because it can be used to estimate the unknown parameters of coupled system. In real life, all the parameters of a system are not known precisely ahead of experiments. The feasibility of the designed controllers is verified and demonstrated numerically and it was found that the coupled system is controlled to equilibrium point when the controllers are activated at time t > 0.

Model description

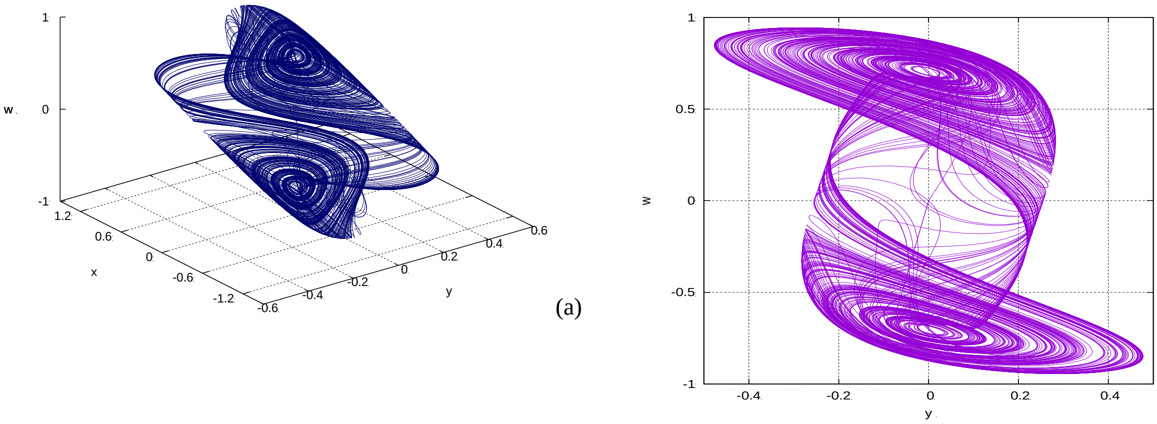
The 4-dimensional memristor-based hyperchaotic system can be described in the dimensionless form by the following set of differential equations (Bao *et al*., 2006):

$$\begin{aligned} \dot{x} &= \gamma a(y - x + dx - W(w)x) \\ \dot{y} &= \gamma(x - y + z) \qquad (1) \\ \dot{z} &= -\gamma(by + cz) \\ \dot{w} &= \gamma x \end{aligned}$$

Where $W(w)$ is the memductance of the memristor and is chosen as:

$$W(w) = \alpha + 3\beta w^2 \qquad (2)$$

Equation (1) governs the 4-dimensional memristor oscillator circuit that has been shown to exhibit rich varieties of dynamical behavior including chaotic motion when the control parameters are respectively chosen as $a = 9.8, b = \frac{100}{7}, c = 0, d = \frac{9}{7}, \alpha = \frac{1}{7}, \beta = \frac{2}{7}, and\ \gamma = 20$.We displayed in Figs. 1 & 2 the phase portrait and the corresponding time series of the chaotic attractor.



(b)

Fig 1: Chaotic attractor of a 4-Dimensional smooth memristor oscillator with the following parameter settings: $a = 9.8, b = \frac{100}{7}, c = 0, d = \frac{9}{7}, \alpha = \frac{1}{7}, \beta = \frac{2}{7}, and\ \gamma = 20.$. (a) **y–x–w** (b) **y–w**.

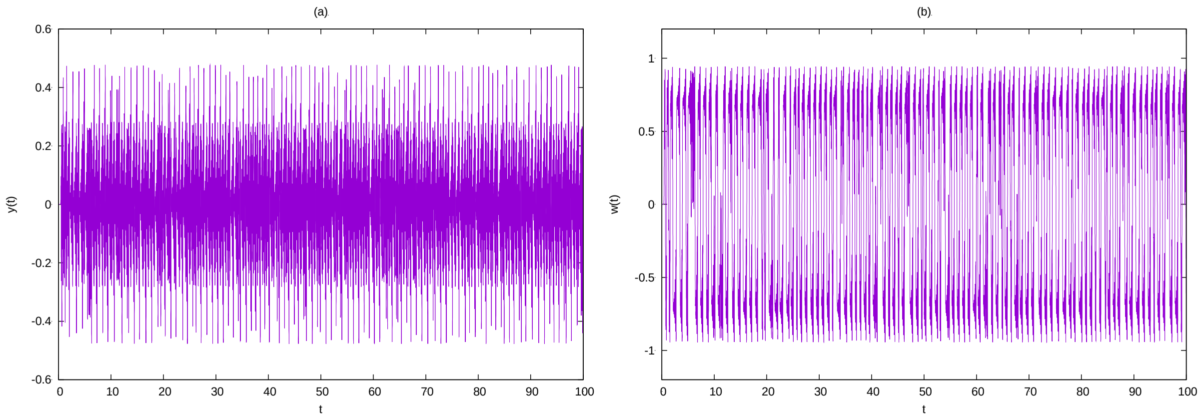


Fig. 2: Time series for hyperchaotic memristive circuit described by Eq. 1 with parameters fixed as in Fig. 1

Design of extended adaptive controllers for controlling chaos in hyperchaotic memristor oscillator

In order to design extended adaptive controllers for the control of hyperchaotic memristor oscillator system, equation (1) is reproduced as follows:

$$\dot{x} = \gamma a(y - x + dx - W(w)x) + u_1(t)$$
$$\dot{y} = \gamma(x - y + z) + u_2(t) \qquad (3)$$
$$\dot{z} = -\gamma(by + cz) + u_3(t)$$
$$\dot{w} = \gamma x + u_4(t)$$

Where $u_i(t)$, $i = 1, 2, 3, 4$, are the nonlinear controllers to be determined later such that the state variables x, y, z, w can be taken to their desired values x_x, y_d, z_d, w_d, respectively.

According to the Lyapunov Stability Theory, we choose the following Lyapunov function:

$$V = \frac{1}{2}\left\{x^2 + y^2 + z^2 + w^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2\right\} \qquad (4)$$

where $\tilde{a} = a - \bar{a}$, $\tilde{b} = b - \bar{b}$, $\tilde{c} = c - \bar{c}$ and $\tilde{d} = d - \bar{d}$. And $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{d}$ are, the estimated values of these unknown parameters respectively.

By differentiating system (4) with respect to time, t, we obtain the following;

$$\dot{V} = x\dot{x} + y\dot{y} + z\dot{z} + w\dot{w} + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} \qquad (5)$$

By substituting equation (3) into equation (5) and letting $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{d}$ take the place of a, b, c, d we obtain the following

$$\dot{V} = x[\gamma\bar{a}(y - x + \bar{d}x - W(w)x) + u_1] + y[\gamma(x - y + z) + u_2] + z[-\gamma(\bar{b}y + \bar{c}z) + u_3] + w[\gamma x + u_4] + \tilde{a}(-\dot{\bar{a}}) + \tilde{c}(-\dot{\bar{c}}) + \tilde{b}(-\dot{\bar{b}}) + \tilde{c}(-\dot{\bar{c}})$$

$$\dot{V} = x[\gamma\bar{a}(y - x + \bar{d}x - W(w)x) + u_1] + y[\gamma(x - y + z) + u_2] + z[-\gamma(\bar{b}y + \bar{c}z) + u_3] + w[\gamma x + u_4] + \tilde{a}(-\dot{\bar{a}} + \gamma x(y - x + \bar{d}x - W(w)x)) + \tilde{d}(-\dot{\bar{d}} + \gamma x^2) + \tilde{b}(-\dot{\bar{b}} - \gamma yz) + \tilde{c}(-\dot{\bar{c}} - \gamma z^2) \quad (6)$$

In order to ensure that the controlled system (3) converges to the equilibrium point $E_0 = (0,0,0,0)$ asymptotically, the following controllers are chosen:

$$u_1 = -\gamma\bar{a}(y - x + \bar{c}x - W(w)x) - x$$
$$u_2 = -\gamma(x - y + z) - y \qquad (7)$$
$$u_3 = \gamma(\bar{b}y + \bar{c}z) - z$$
$$u_4 = -\gamma x - w$$

And the following parameter estimation update laws are chosen:

$$\dot{\bar{a}} = \gamma x(y - x + \bar{c}x - W(w)x) + \tilde{a}$$
$$\dot{\bar{c}} = \gamma x^2 + \tilde{c} \qquad (8)$$
$$\dot{\bar{b}} = -\gamma yz + \tilde{b}$$
$$\dot{\bar{d}} = \gamma z^2 + \tilde{d}$$

Substituting equation (7) and (8) into equation (6) one readily obtains:

$\dot{V} = -x^2 - y^2 - z^2 - w^2 - \tilde{a}^2 - \tilde{c}^2 - \tilde{b}^2 - \tilde{d}^2 < 0$(9)

According to the Lyapunov stability theory, the condition defined by equation (9) ensures that the controlled system (3) converges to the equilibrium point with the controllers in equation (7) and the parameter estimation update law described by system (8).

Numerical simulations

To verify the effectiveness and feasibility of the controllers obtained in equations (8) and (9), fourth-order Runge-Kutta algorithm is employed with initial conditions $(0, 10^{-10}, 0, 0)$, a time step of 0.001 and fixing the parameter values as in Fig. 1 to ensure chaotic dynamics of the coupled systems. The result obtained showed that for time $t \leq 50$, the dynamics of the state variables move chaotically with time when the control functions defined in equation (7) is deactivated as shown in Fig. 3.

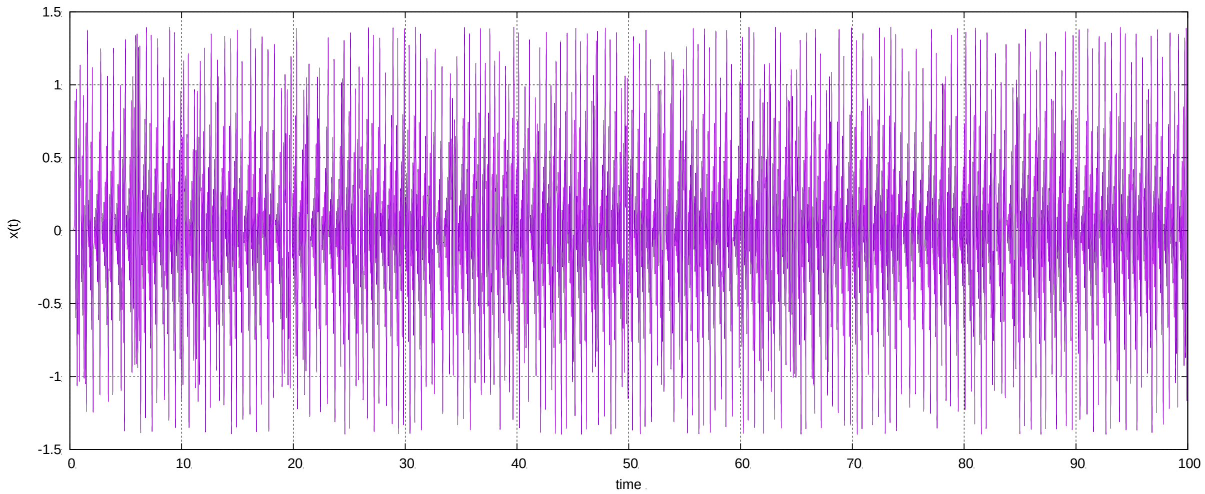
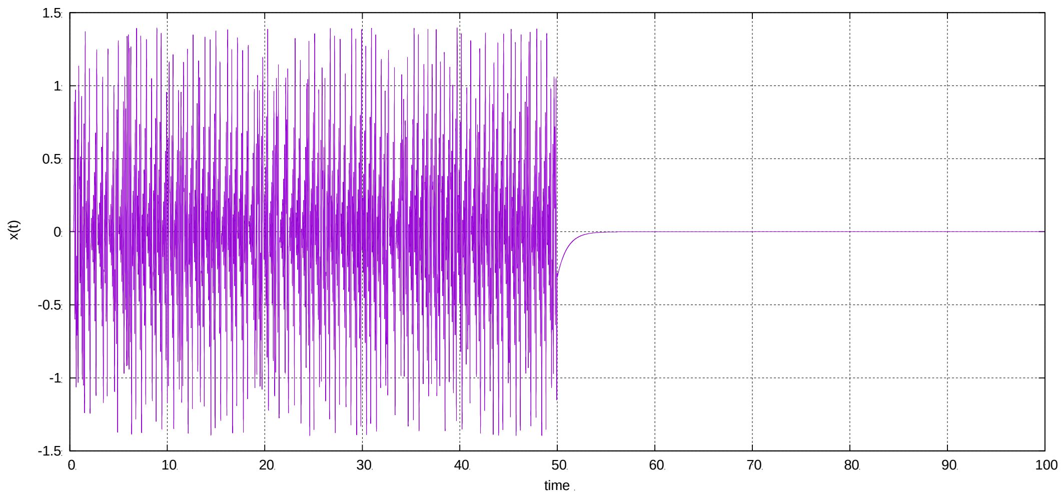
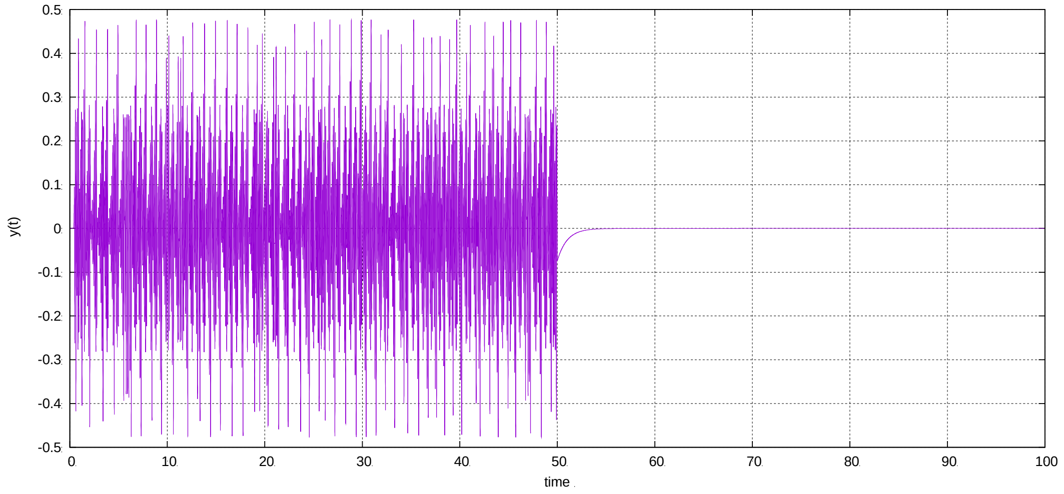


Fig. 3: Time series of hyperchaotic memristor oscillator circuit without controller

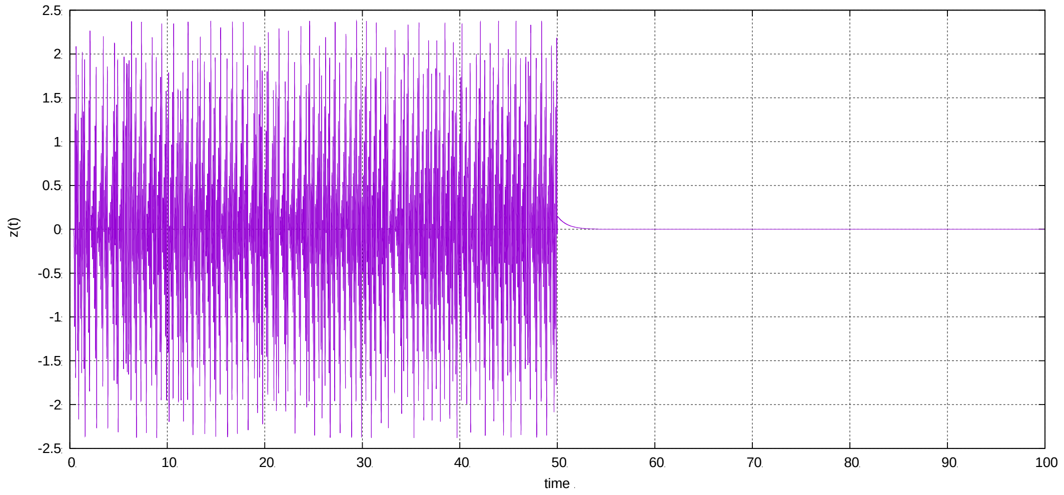
With the controllers activated at $t \geq 50$, the state variables were controlled to the origin as shown in Fig. 4.



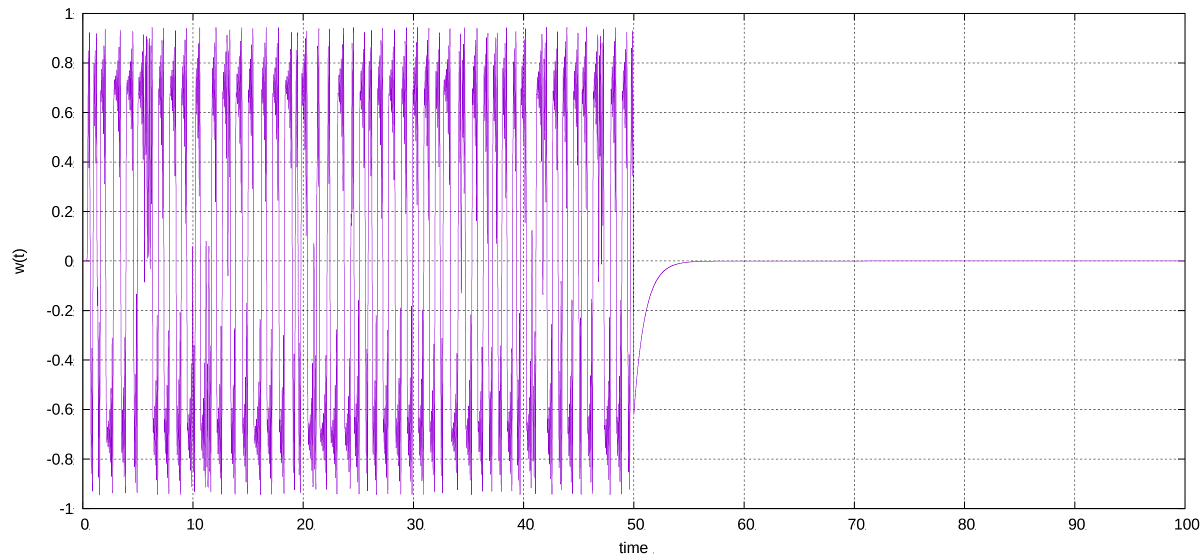
(a)



(b)



(c)



(d)

Figure 4: (a) – (d) Adaptive controller tracking control of hyperchaotic memristor oscillator when the controller is activated at t = 50.

Design of extended adaptive controller for synchronization of chaos in hyperchaotic memristor oscillators

In order to achieve synchronization between two 4-dimensional memristor oscillator circuits evolving from different initial conditions, drive system and the response system are respectively given as:

$$\dot{x}_1 = \gamma a(y_1 - x_1 + cx_1 - (\alpha + 3\beta w_1^2)x_1)$$
$$\dot{y}_1 = \alpha(x_1 - y_1 + z_1) \qquad (10)$$
$$\dot{z}_1 = -\gamma(by_1 + cz_1)$$

$\dot{w}_1 = \gamma x_1$
And;
$\dot{x}_2 = \gamma a(y_2 - x_2 + cx_2 - (\alpha + 3\beta w_2{}^2)x_2) + u_1$
$\dot{y}_2 = \alpha(x_2 - y_2 + z_2) + u_2$ (11)
$\dot{z}_2 = -\gamma(by_2 + cz_2) + u_3$
$\dot{w}_2 = \gamma x_2 + u_4$
Where: $u_i(t)$, $i = 1, 2, 3, 4$, are the nonlinears controllers to be determined later.
Let the error states between the state variables of the response and drive systems be defined as follows:
$e_x = x_2 - x_1$; $e_y = y_2 - y_1$; $e_z = z_2 - z_1$; $e_w = w_2 - w_1$ (12)

Subtracting equation (10) from equation (11), the following error dynamical system is obtained:
$\dot{e}_x = \gamma a(e_y - e_x + ce_x - \alpha e_x - 3\beta[w_2{}^2x_2 - w_1{}^2x_1]) + u_1$
$\dot{e}_y = \gamma(e_x - e_y + e_z) + u_2$ (13)
$\dot{e}_z = -\gamma(be_y + ce_z) + u_3$
$\dot{e}_w = \gamma e_x + u_4$
The following Lyapunov function is chosen:
$V = \frac{1}{2}(e_x{}^2 + e_y{}^2 + e_z{}^2 + e_w{}^2 + \tilde{a}^2 + \tilde{c}^2 + \tilde{b}^2 + \tilde{d}^2)$(14)
Where: $\tilde{a} = a - \bar{a}, \tilde{b} = b - \bar{b}, \tilde{c} = c - \bar{c}, \tilde{d} = d - \bar{d}.$
The parameters $\bar{a}, \bar{b}, \bar{c}, and\ \bar{d}$ are the estimated values of these unknown parameters, respectively.

By differentiating (13) with respect to time, the following is obtained
$\dot{V} = e_x\dot{e}_x + e_y\dot{e}_y + e_z\dot{e}_z + e_w\dot{e}_w + \tilde{a}\dot{\tilde{a}} + \tilde{c}\dot{\tilde{c}} + \tilde{b}\dot{\tilde{b}} + \tilde{d}\dot{\tilde{d}}$ (15)
By substituting equation (12) into equation (14) and letting $\bar{a}, \bar{b}, \bar{c}, and\ \bar{d}$ take the place of a, b, $c\ and\ d$ one readily obtains the following:
$$\dot{V} = e_x(\gamma\bar{a}(e_y - e_x + \bar{c}e_x - \alpha e_x - 3\beta[w_2{}^2x_2 - w_1{}^2x_1]) + u_1) + e_y(c(e_x - e_y + e_z) + u_2) + e_z(-c(\bar{b}e_y + \bar{d}e_z) + u_3) + e_w(\gamma e_x + u_4) + \tilde{a}(-\dot{\tilde{a}}) + \tilde{c}(-\dot{\tilde{c}}) + \tilde{b}(-\dot{\tilde{b}}) + \tilde{c}(-\dot{\tilde{c}})$$

$$\dot{V} = e_x(\gamma\bar{a}(e_y - e_x + \bar{c}e_x - \alpha e_x - 3\beta[w_2{}^2x_2 - w_1{}^2x_1]) + u_1) + e_y(\gamma(e_x - e_y + e_z) + u_2) + e_z(-\gamma(\bar{b}e_y + \bar{c}e_z) + u_3) + e_w(\gamma e_x + u_4) + \tilde{a}(-\dot{\tilde{a}} + \gamma e_x(e_y - e_x + \bar{c}e_x - \alpha e_x - 3\beta[w_2{}^2x_2 - w_1{}^2x_1])) + \tilde{c}(-\dot{\tilde{c}} + \gamma e_x{}^2) + \tilde{\beta}(-\dot{\tilde{\beta}} + (-`\gamma e_y e_z)) + \tilde{c}(-\dot{\tilde{c}} + (-\gamma e_x{}^2)) \quad (16)$$

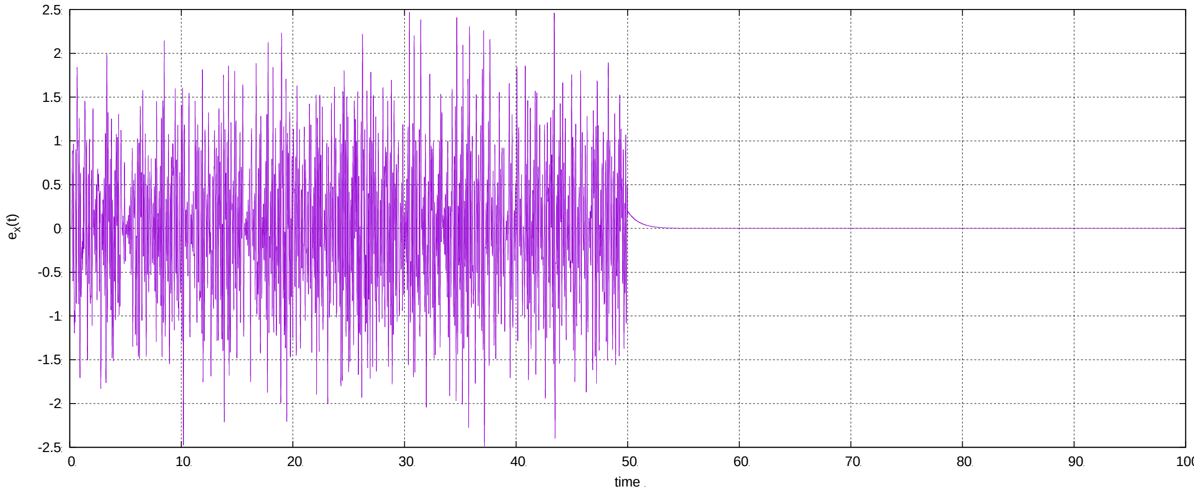
In order to ensure that the error dynamical system (13) converges to the origin asymptotically, the condition $\dot{V} < 0$ must be satisfied. From equation (16) the following controllers are selected:
$u_1(t) = -\gamma\bar{a}(e_y - e_x + \bar{c}e_x - \alpha e_x - 3\beta[w_2{}^2x_2 - w_1{}^2x_1]) - e_x$
$u_2(t) = -\gamma(e_x - e_y + e_z) - e_y$ (17)
$u_3(t) = \gamma(\bar{b}e_y + \bar{c}e_z) - e_z$
$u_4(t) = -\gamma e_x - e_w$
And the parameter update laws are chosen as follows:
$\dot{\bar{a}} = \gamma e_x(e_y - e_x + \bar{c}e_x - \alpha e_x - 3\beta[w_2{}^2x_2 - w_1{}^2x_1]) + \tilde{a}$
$\dot{\bar{c}} = \gamma e_x{}^2 + \tilde{c}$ (18)
$\dot{\bar{b}} = -\gamma e_y e_z + \tilde{b}$
$\dot{\bar{d}} = -\gamma e_x{}^2 + \tilde{d}$
Substituting equation (17) and (18) into equation (16) the following equation is obtained:
$\dot{V} = -x^2 - y^2 - z^2 - w^2 - \tilde{a}^2 - \tilde{c}^2 - \tilde{b}^2 - \tilde{d}^2 < 0$ (19)
With condition (19) the error dynamical system converges to the origin asymptotically in line with the Lyapunov stability theory. Also the drive system (11) is synchronized with the response system (12) with controller (17) and the parameter update law (18).

Numerical simulations

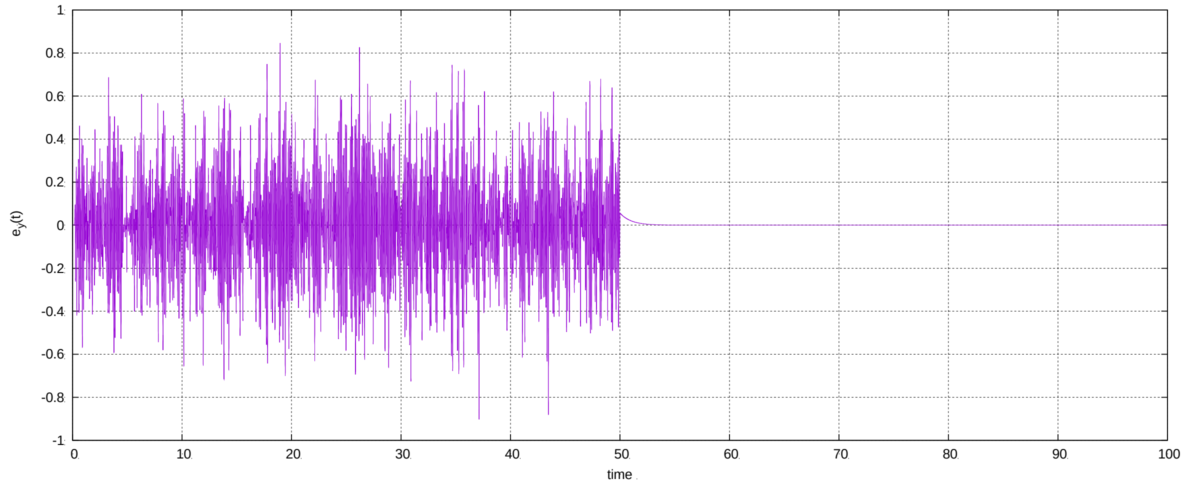
By fixing the parameter values as in figure 1 to ensure chaotic dynamics of the state variables, systems (11) and (12) were solved with the control function as defined in equation (17). The result obtained shows that the error state variable moved hyperchaotically in time when the controller is switched off and when the controller is activated at t = 50 (see Figure 5), the error state variable converges to zero, thereby guaranteeing the asymptotic stability of the systems (11) and (12). This is defined by the synchronization quality e given as (Pecora and Carroll, 1990).

$$e = \sqrt{e_x{}^2 + e_y{}^2 + e_z{}^2 + e_w{}^2}$$

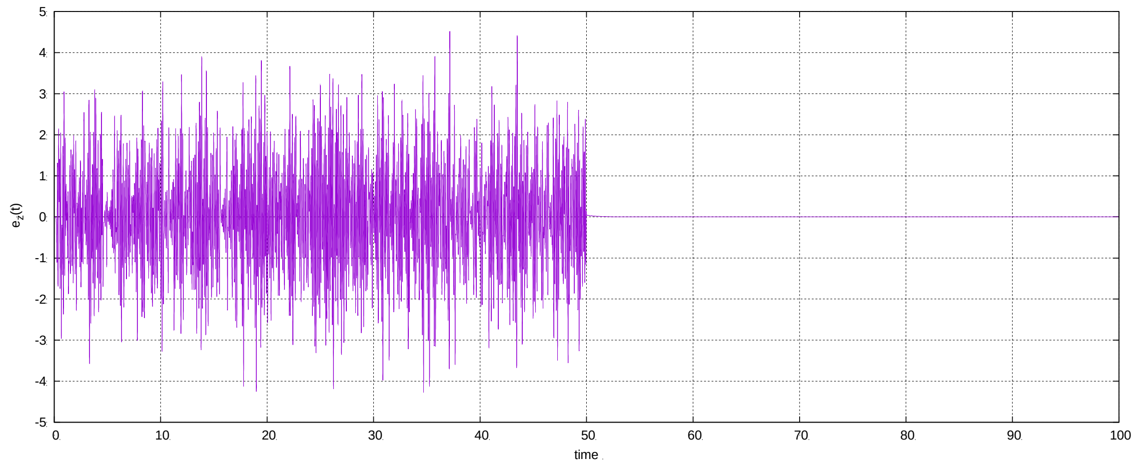
In Fig. 6, we display the convergence of response system to drive one after switching controllers on at time $t = 50$. Again, this shows the effectiveness of the designed controllers. The initial values of the parameter update laws (18) are chosen as $a_1(0) = -0.5$, $b_1(0) = 3.0$, $c_1(0) = 2.0$ and $d_1(0) = -5.0$. The parameter estimation values $\bar{a}$, $\bar{b}$, $\bar{c}$ and $\bar{d}$ converges to $a = 9.8, b = \frac{100}{7}, c = 0, d = \frac{9}{7}$, respectively as shown in Fig. 7.



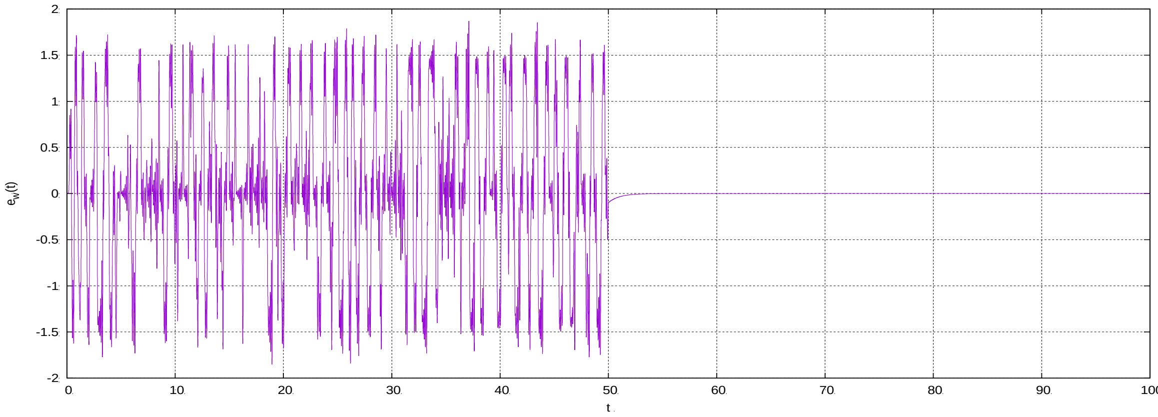
(a)



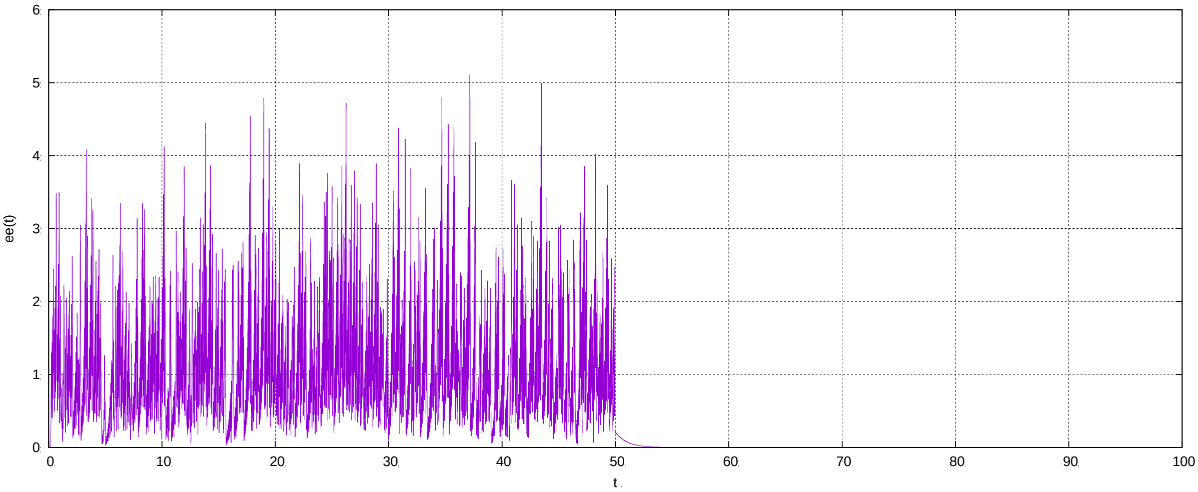
(b)



(c)

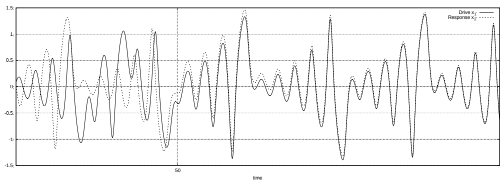


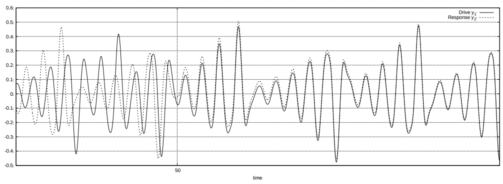
(d)

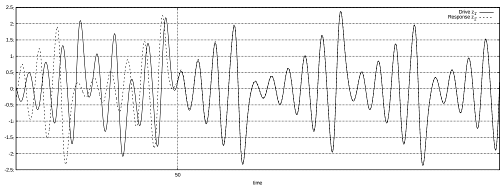


(e)

Figure 5: (a) – (e) Error dynamics between the two hyperchaotic memristor oscillator circuit with extended adaptive controllers deactivated for $0 < t < 50$ and activated for $t \geq 50$.


(a)


(b)


(c)

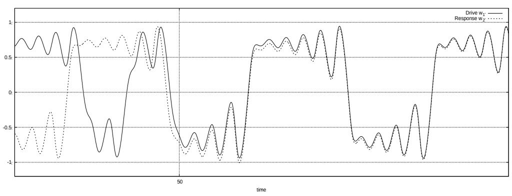

(d)

Fig. 6: (a) – (e) Convergence of 4-dimensional response system to drive after switching controllers on at time $t = 50$.

Time Series of corresponding variables $(x_1, x_2), (y_1, y_2), (z_1, z_2)$ and (w_1, w_2) showan intermediate phase synchronization between the drive and response system before the attainment of complete synchronization.

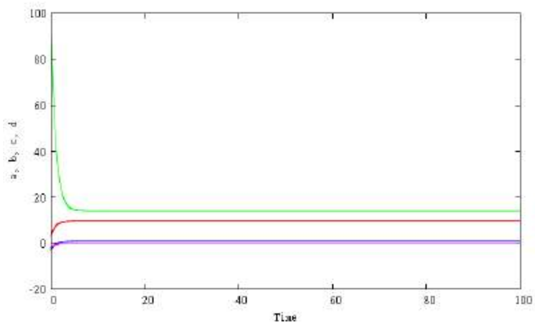


Fig. 7: Time response of parameter estimation errors

Conclusion

In this work, the adaptive control techniques have been applied to control and synchronize hyperchaotic memoristive system with unknown parameters. The designed controllers were found to be very effective to control chaotic behaviour and globally synchronize two identical memoristive systems evolving from different initial conditions. Numerical simulations are given to demonstrate the effectiveness of the proposed controllers. Control and synchronization of memoristive system suggests the possibility for communication using chaotic wave forms as carriers, perhaps with application to secure communication. Thus, practical implementation of the proposed scheme shall be very useful and the future work shall focus on addressing this problem.

Acknowledgement

The authors sincerely thanked the anonymous reviewers for their useful comments and constructive criticism that really help to improve the quality of the work.

References

Aguilar-López R, Martnez-Guerra R & Perez-Pinacho C 2014. Nonlinear observer for synchronization of chaotic systems with application to secure data transmission. *European Physical J. Special Topics,* 223, 1541–1548.

Bao BC, Liu Z & Xu JP 2010. Transient chaos in smooth memristor oscillator, *Chinese Physics* B, 19(3): 030510-1 - 030510-6.

Boutayeb M, Daroach M& Rafaralahy H 2002.Generalized state-space observers for chaotic synchronization and secure communication. *IEEE Trans. Circuits. Syst. I,* 49(3): 345–349.

Cuomo KM, Oppenheim AV & Strogatz SH 1993. Synchronization of Lorenz-based chaotic circuits with applications to communications. *IEEE Trans. Circuits Syst. II, Express Briefs,* 40: 626–633.

Di LC, Liao XF & Wong DW 2005. Lag synchronization of hyperchaos with application to secure communications. *Chaos Solitons & Fractals,* 23(1): 183–193.

Elabbasy EM, Agiza HN & Dessoky MM 2006. Adaptive synchronization of a hyperchaotic system with uncertain parameter. *Chaos Solitons & Fractals,* 30(5): 1133.

Eisencraft M, Fanganiello RD, Grzybowski JMV, Soriano DC, Attux R, Batista AM, Macau EEN, Monteiro LHA, Romano JMT, Suyama R & Yoneyama T 2012. Chaos-based communication systems in non-ideal channels. *Commun. Nonlinear Sci. & Numer. Simul.,* 17(12): 4707–4718.

Feki M 2003. An adaptive chaos synchronization scheme applied to secure communication. *Chaos Solitons & Fractals,*18(1): 141–148.

Filali RL, Benrejeb M & Borne P 2014. On observer-based secure communication design using discrete-time hyperchaotic systems.*Commun.Nonlinear Sci & Numer.Simul.*,19(5): 1424–1432.

Guan XP, Fan ZP & Chen CL 2002.Chaos Control and Its Application in Secure Communication.*National Defence Industry Press, Beijing*, pp. 1-37.

Ho MC, Hung YC & Chou CH 2002.Phase and anti-phase synchronization of two chaotic systems by using active control. *Physics Letters,* A 296(1): 43–48.

Liu YJ, Tong SC, Wang W & Li YM 2009. Observer-based direct adaptive fuzzy control of uncertain nonlinear systems and its applications. *Int. J. Control Automation Sys.,* 7(4): 681 – 690.

Ma DZ, Zhang HG, Wang ZS & Feng J 2010. Fault tolerant synchronization of chaotic systems based on T-S fuzzy model with fuzzy sampled-data controller. *Chinese Physics,* B19(5): 050506.

Mainieri R & Rehacek J 1999. Projective synchronization in three-dimensional chaotic systems.*Physical Review Letters,* 82: 3042–3045.

Michael GR, Arkady SP & Jurgen K 1996. Phase synchronization of chaotic oscillators. *Physical Review Letters,* 76(11): 1804–1807.

Olusola O I, Vincent UE & Njah AN 2009. Global chaos synchronization of coupled parametrically excited pendula. *Pramana Journal,* 73: 1011–1022.

Pan L, Zhou WN, Fang A & Li DQ 2010. A novel active pinning control for synchronization and anti-synchronization of new uncertain unified chaotic systems. *Nonlinear Dynamics,* 62(1–2): 417–425.

Pecora LM, Carroll TL 1990. Synchronization in chaotic system. *Physical Review Letters,* 64(8): 821 – 824.

Pikovsky A, Rosenblum M & Kurths J 2001. Synchronization: A Universal Concept in Non-Linear Sciences. *Cambridge University Press, United Kingdom*, pp. 1 – 354.

Ren HP, Baptista MS & Grebogi C 2013. Wireless communication with chaos. *Physical Review Letters,* 110: 184101 - 184104.

Sundar S & Minai AA 2000. Synchronization of randomly multiplexed chaotic systems with applications to communication. *Physical Review Letters,* 85(25): 5456–5459.

Wang YW & Guan ZH 2006. Generalized synchronization of continuous chaotic system. *Chaos Solitons & Fractals,* 27(1): 97–101.

XingyuanWang N & YaqinWang J 2011. Adaptive control for synchronization of a four-dimensional chaotic system via a single variable. *Nonlinear Dynamics,* 65: 311–316.

Yang SS & Duan CK 1998. Generalized synchronization in chaotic systems.*Chaos Solitons & Fractals,* 9(10): 1703–1707.

Ying T& Chu LO 1997.Impulsive control and synchronization of nonlinear dynamical systems and application to secure communication.*Int. J. Bifurcation & Chaos,* 7(3): 645–667.

Zhang HG, Ma TD, Huang GB & Wang ZL 2010. Robust global exponential synchronization of uncertain chaotic delayed neural networks via dual-stage impulsive control. *IEEE Transactions on Systems, Man & Cybernetics, Part B,* 40(3): 831–844.

Zhang Y & Sun J 2004. Chaotic synchronization and anti-synchronization based on suitable separation. *Physics Letters,* A330(6): 442–447.